16.4: Integrals in Polar Coordinates

Calculus III

College of the Atlantic. Winter 2016

1. Consider the integral $\int_R f(r,\theta)\,dA$. Write the integral as an iterated integral for the following regions R:

   (a) R is a circle of radius 7, centered at the origin.

   (b) R is the top half of the unit circle.

   (c) R is the left half of the unit circle.

   (d) R is an annulus (a flat donut) centered at the origin with inner radius 1 and outer radius 3.

2. Repeat the above question, but express the iterated integrals as integrals in x and y instead of r and θ.

3. Evaluate the following integrals:

$$\int_Q \frac{1}{x^2+y^2}\,dA \tag{1}$$

$$\int_Q y\,dA \tag{2}$$

   Where Q is the region bounded by $y=\sqrt{1-x^2}$, $y=\sqrt{9-x^2}$, and the positive x and y axes.

4. Convert the following integral to polar coordinates and evaluate it:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\,dx\,dy \tag{3}$$