16.4 and 16.5: Integrals in Polar, Cylindrical, and Spherical Coordinates

Calculus III

College of the Atlantic. Winter 2016

1. Convert the following integral to polar coordinates and evaluate it:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2-y^2}\, dx\, dy \tag{1}$$

2. Sketch or describe the following surfaces:
   (a) In cylindrical coordinates:
      i. $z = 3$
      ii. $\theta = \pi/6$
      iii. $\theta = \pi$
      iv. $r = 4$
   (b) In spherical coordinates:
      i. $\rho = 4$
      ii. $\theta = \pi/6$
      iii. $\theta = \pi$
      iv. $\phi = \pi/6$
      v. $\phi = \pi/2$

3. Set up a triple integral for a density function integrated over the first octant of a sphere of radius 9.

4. Set up a triple integral for a density function integrated over a cylinder with radius 5 and height 10.

5. Set up a triple integral for a density function integrated over a cone with a radius of 9 and a height of 9.

6. Set up a triple integral for a density function integrated over the eighth octant of a sphere of radius 9 (i.e., the octant in which x is positive, y and z are negative.)

7. Evaluate the above integrals assuming that the density function is constant: $f(x, y, z) = k$.

8. Casey is eating a wedge of cheese. The wedge was taken from a cylinder with a radius of 7 cm. The height of the cheese is 3 cm. The angle at the tip of the wedge is 30 degrees. The yumminess density of the cheese, in units of yumminess per cubic cm, is proportional to the distance from the edge. (I.e., the middle of the cheese wheel is tastier.) Write an iterated triple integral for the total yumminess of the wedge of cheese.