

17.1 and 17.2: Parametrized Curves and their Derivatives

Calculus III

College of the Atlantic. Winter 2016

Sketch or describe the following curves:

1. $[4, -2, 5]$
2. $[4t, -2, 5]$
3. $[4t, -2t, 5t]$
4. $[\cos(t), \sin(t), 0]$
5. $[\cos(2t), \sin(2t), 0]$
6. $[\cos(20t), \sin(20t), 0]$
7. $[\cos(t), \sin(t), t]$
8. $[\cos(t), \sin(t), 2]$
9. $[\cos(t), \sin(2t), 0]$
10. $[\cos(t), \sin(3t), 0]$
11. $[t \cos(t), t \sin(t), 0]$
12. $[2t \cos(t), t \sin(t), 0]$
13. $[\cos(t), \sin(t), \cos(t)]$
14. $[\cos(t), \sin(t), \cos(3t)]$
15. $[\sin(t) + 2 \sin(4t), \cos(t) - 2 \cos(4t), -\sin(3t)]$

Write parametrized curves for the following:

1. A line parallel to $2\hat{i} + 3\hat{j} + 4\hat{k}$ and through the point $(1, 5, 7)$.
2. A line from $(0, 0)$ to $(0, 4)$
3. A quarter circle with radius 2 in the first quadrant, moving counter-clockwise.
4. A line from $(4, 0)$ to $(0, 0)$.

1. The position of a rabbit is given by $r(\vec{t}) = (3 \cos(2t), 3 \sin(2t))$.

(a) Sketch the motion of the bird.

(b) Find the bird's velocity vector $v(\vec{t})$.

(c) Sketch $v(\vec{0})$ and $v(\vec{\pi}/2)$.

(d) Find the bird's acceleration vector $a(\vec{t})$.

(e) Show that $\|\vec{a}\| = \|\vec{v}\|^2/r$.

2. A TAB mug is thrown from a rooftop at time $t = 0$ seconds. Its position at time t is given by

$$r(\vec{t}) = 10t\hat{i} - 5t\hat{j} + (6.4 - 4.9t^2)\hat{k}. \quad (1)$$

The origin is the base of the building, which stands on flat ground. The vector \hat{i} points east, \hat{j} points north, and \hat{k} points up.

(a) How high is the rooftop above the ground?

(b) At what time does the soil hit the ground?

(c) How fast is the soil moving when it hits the ground?