Worksheet 12.1: Functions of Two Variables

1. Let $C = f(I, p)$ represent the consumption of beef (in pounds per week per household) by a family whose household income is I, in thousands of dollars, when the price of beef is p dollars per pound. Values of the function f are shown in the table:

Household income per year, I (1000)	Price of beef, ($/lb)			
	3.00	3.50	4.00	4.50
20	2.65	2.59	2.51	2.43
40	4.14	4.05	3.94	3.88
60	5.11	5.00	4.97	4.84
80	5.35	5.29	5.19	5.07
100	5.79	5.77	5.60	5.53

(a) Explain the meaning of the statement $f(60, 3.50) = 5$ in practical terms.

(b) Solve each of the following equations for the unknown variable, and give a practical interpretation of your answer.
   (i) $f(80, x) = 5.19$
   (ii) $f(x, 3) = 2.65$

(c) Is $C = f(60, p)$ an increasing or a decreasing function of p? Explain why this makes sense in the context of this problem.

(d) Is $C = f(3.5, I)$ an increasing or a decreasing function of I? Explain why this makes sense in the context of this problem.

2. Two people standing 10 feet apart are holding opposite ends of a jump rope. The height of the jump rope x feet from the person on the left is given by

$$h(x, t) = 3 + 3 \sin\left(\frac{\pi}{10}x\right) \cos(2\pi t),$$

where t represents time in seconds.

(a) Sketch graphs of h versus x for the following fixed values of t: $0, 1/8, 1/4$.

(b) For what fixed values of t is the function $h(x, t)$ a constant function? Explain what is happening with the jump rope at these times.

(c) Sketch a graph of the function $h(2, t)$. What does this function represent?

(d) Give a practical interpretation of each of the following quantities:
   (i) $h(5, 2) - h(0, 2)$
   (ii) $h(5, 2) - h(5, 0)$