

# Functions of Two Variables

## Calculus III

College of the Atlantic

On a separate sheet of paper is a numerical representation of a function of two variables. Let's call the two variables  $x$  and  $y$  and we'll call the function  $f(x, y)$ .

1. What would a plot of this function look like? Make a rough sketch.
2. At  $x = 1$  and  $y = 1$ , is the function increasing in the  $x$  direction? Is it increasing in the  $y$  direction?
3. At  $x = -1$  and  $y = 1$ , is the function increasing in the  $x$  direction? Is it increasing in the  $y$  direction?
4. At  $x = 1$  and  $y = -1$ , is the function increasing in the  $x$  direction? Is it increasing in the  $y$  direction?
5. At  $x = -1$  and  $y = -1$ , is the function increasing in the  $x$  direction? Is it increasing in the  $y$  direction?
6. Consider all the  $x, y$  pairs for which  $f(x, y) = 1$ . What does the set of these  $x, y$  pairs look like? Make a sketch.
7. Consider all the  $x, y$  pairs for which  $f(x, y) = 2$ . What does the set of these  $x, y$  pairs look like? Make a sketch.
8. Consider all the  $x, y$  pairs for which  $f(x, y) = 3$ . What does the set of these  $x, y$  pairs look like? Make a sketch.

9. Sketch and interpret, in words, the following:
- (a)  $f(1, y)$
  - (b)  $f(2, y)$
  - (c)  $f(x, -1)$
  - (d)  $f(x, 0)$
10. What is the meaning of the following quantities:
- (a)  $f(0, 0)$
  - (b)  $f(2, -1)$
11. By staring at the numbers, guess a formula for the function  $f(x, y)$ .
12. Using this formula, come up with algebraic answers to questions 6–8.
13. Using this formula, come up with algebraic answers to question 9.

### **Distances and Spheres:**

1. How far is the point  $(3, 4, 5)$  from the origin?
2. How far is the point  $(3, 4, 5)$  from the point  $(1, 2, 3)$ ?
3. Write down the equation of a sphere with radius 5 centered at the origin.
4. Write down the equation of a sphere with radius 5 centered at the point  $(2, -2, 5)$ .