16.1: Integrals in One and Two Dimensions

Calculus III

College of the Atlantic

1. Unicorns live along a 100-mile stretch of a path through unicorn valley. The UCD[1] has determined that the rate of occurrence of unicorns is given in the table below, in units of unicorns per mile.
   (a) Find upper and lower estimates for the total number of unicorns that live along the path.
   (b) Suppose you knew that the unicorn density was described by the function $\rho(x)$. Write an exact expression for the number of unicorns that live along the path.
   (c) Determine an approximate expression for the average density of unicorns that live along the path.
   (d) Write an exact expression for average density of unicorns that live along the path in terms of the density function $\rho(x)$.

2. In a region of the UK, someone has compiled remarkably detailed information about the spatial distribution of foxes. The map in Fig. 1 shows contour lines for fox density, measured in foxes per square kilometer. Estimates for the density in each square on the map are found in Fig. 2. These estimate were formed by using the approximate density value in the northeast corner of each box.
   (a) Write an approximate expression for the total number of foxes.
   (b) Suppose that the spatial density of foxes is known to be given by the function $\sigma(x, y)$, in units of foxes per square kilometer. Write down an exact expression for the total number of foxes.
   (c) Write down an exact expression for the average fox density.

3. The height of a surface is given by the function $z = x^2y$. What is the of the volume between the surface and sea level ($z = 0$), above the rectangular region R whose corners are: $(0, 0)$, $(20, 0)$, $(0, 10)$, and $(20, 10)$. Do not use the fundamental theorem of calculus. Instead, make a table of numbers with $\Delta x = 5$ and $\Delta y = 5$.

4. Express the above integrals as iterated integrals, and evaluate them.

Mile	Unicorn Density Density
0	8
25	4
50	3
75	3
100	0

[1] Unicorn Census Department

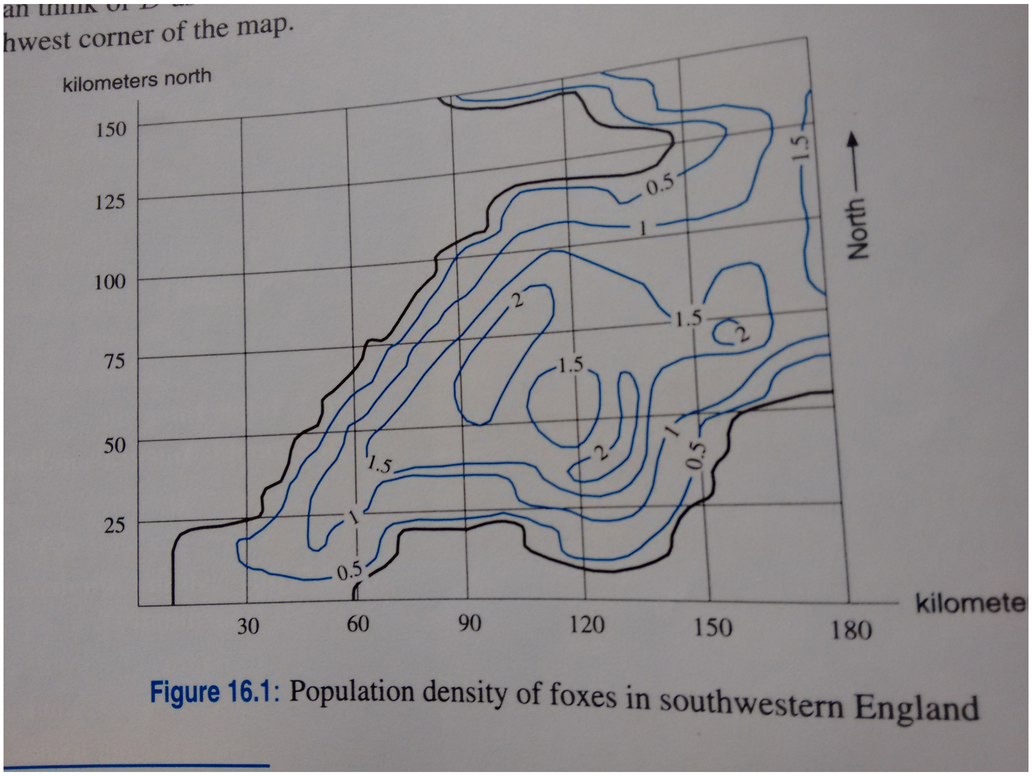
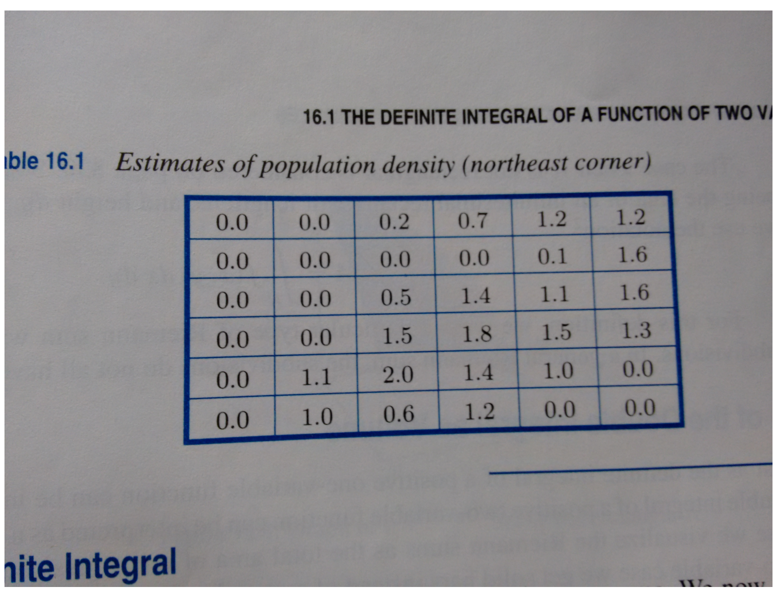


Figure 1: A contour map of fox densities.



16.1 THE DEFINITE INTEGRAL OF A FUNCTION OF TWO V

ble 16.1 *Estimates of population density (northeast corner)*

0.0	0.0	0.2	0.7	1.2	1.2
0.0	0.0	0.0	0.0	0.1	1.6
0.0	0.0	0.5	1.4	1.1	1.6
0.0	0.0	1.5	1.8	1.5	1.3
0.0	1.1	2.0	1.4	1.0	0.0
0.0	1.0	0.6	1.2	0.0	0.0

nite Integral

Figure 2: A table of fox densities.