16.4: Integrals in Polar Coordinates

Calculus III

College of the Atlantic

1. Let the density of butterflies be given by $\rho(x, y, z)$. Write down an iterated integral that gives the total number of butterflies in a sphere of radius 3 centered on the origin.

2. Consider the integral $\int_R f(r, \theta)\, dA$. Write the integral as an iterated integral for the following regions R:

   (a) R is a circle of radius 7, centered at the origin.

   (b) R is the top half of the unit circle.

   (c) R is the left half of the unit circle.

   (d) R is an annulus (a flat donut) centered at the origin with inner radius 1 and outer radius 3.

3. Repeat the above question, but express the iterated integrals as integrals in x and y instead of r and θ.

4. Evaluate the following integrals:

$$\int_Q \frac{1}{x^2 + y^2}\, dA \tag{1}$$

$$\int_Q y\, dA \tag{2}$$

   Where Q is the region bounded by $y = \sqrt{1 - x^2}$, $y = \sqrt{9 - x^2}$, and the positive x and y axes.

5. Convert the following integral to polar coordinates and evaluate it:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2 - y^2}\, dx\, dy \tag{3}$$