16.5: Integrals in Polar and Cylindrical and Spherical Coordinates

Calculus III

College of the Atlantic

1. Evaluate

$$\int_Q y \, dA \tag{1}$$

   Where Q is the region bounded by $y = \sqrt{1 - x^2}$, $y = \sqrt{9 - x^2}$, and the positive x and y axes.

2. Convert the following integral to polar coordinates and evaluate it:

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-x^2 - y^2} \, dx \, dy \tag{2}$$

3. Sketch or describe the following surfaces in cylindrical coordinates:

   (a) $z = 3$

   (b) $\theta = \pi/6$

   (c) $\theta = \pi$

   (d) $r = 4$

4. Set up a triple integral for a density function integrated over a cylinder with radius 5 and height 10.

5. Set up a triple integral for a density function integrated over a cone with a radius of 16 and a height of 9.

6. Sketch or describe the following surfaces in spherical coordinates:

   (a) $\rho = 4$

   (b) $\theta = \pi/6$

   (c) $\theta = \pi$

   (d) $\phi = \pi/6$

   (e) $\phi = \pi/2$

7. Set up a triple integral for a density function integrated over the first octant of a sphere of radius 9.

8. Set up a triple integral for a density function integrated over the eighth octant of a sphere of radius 9 (i.e., the octant in which x is positive, y and z are negative.)