

17.2: Parametrized Curves and their Derivatives

Calculus III

College of the Atlantic

1. The position of a butterfly is given by $r(\vec{t}) = (3 \cos(2t), 3 \sin(2t))$.

- Sketch the motion of the butterfly.
- Find the butterfly's velocity vector $v(\vec{t})$.
- Sketch $v(\vec{0})$ and $v(\vec{\pi}/2)$.
- Find the butterfly's acceleration vector $a(\vec{t})$.
- Show that $\|\vec{a}\| = \|\vec{v}\|^2/r$.
- Show that $\vec{a} \perp \vec{v} = 0$.

2. A TAB mug is thrown from a rooftop at time $t = 0$ seconds. Its position while in the air at time t is given by

$$r(\vec{t}) = 10t\hat{i} - 5t\hat{j} + (6.4 - 4.9t^2)\hat{k}. \quad (1)$$

The origin is the base of the building, which stands on flat ground. The vector \hat{i} points east, \hat{j} points north, and \hat{k} points up.

- How high is the rooftop above the ground?
- At what time does the mug hit the ground?
- How fast is the mug moving when it hits the ground?