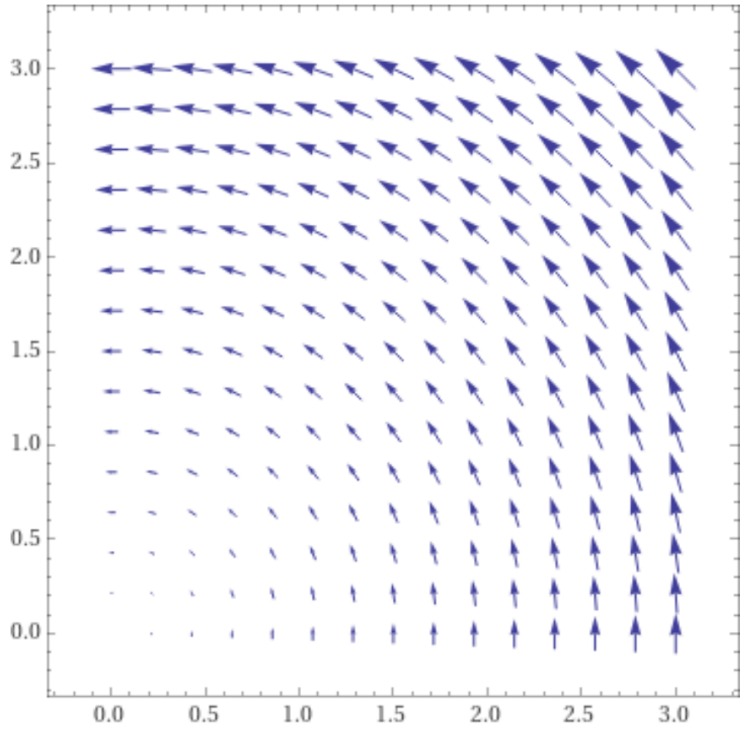
18.2: More Evaluating Line Integrals

Calculus III

College of the Atlantic



Consider the vector field,

$$\vec{F}(x,y) = -y\vec{i} + x\vec{j}\,, \tag{1}$$

plotted in the figure above. In class last time we found that

$$\int_{C_1} \vec{F} \cdot d\vec{r} = 4\,, \tag{2}$$

where C_1 is the line segment that starts at $(2,0)$ and ends at $(0,2)$.

1. Today we will investigate the line integral along the curve C_2, where C_2 is the quarter-circle of radius 2 centered at the origin. First, write a parametrization for the curve.

2. Btw, for $\vec{r(t)}$, what is $\vec{r'(t)}$ at $t = 0$, $t = \pi/4$, and $t = \pi/2$? Draw the curve and draw the velocity vectors at these three points.

3. Without evaluating the integral, do you think

$$\int_{C_2} \vec{F} \cdot d\vec{r} \tag{3}$$

   is greater than or less than 4?

4. Now evaluate the integral.

5. Could $\vec{F}$ be a gradient field?