20.1: Divergence

Calculus III

College of the Atlantic

1. The density of air in a room is given by $f(x, y, z) = z + xy^2$. What is the mass of air in a sphere of radius 0.01 centered at $(1, 2, 3)$?

2. A vector field has a divergence $\nabla \cdot \vec{F} = z + xy^2$ in m^3/s. What is the flux flowing out of a sphere of radius 0.01 centered at $(1, 2, 3)$?

3. Find the divergence of the following vector fields:

$$\vec{F} = y\hat{\imath} - x\hat{\jmath} \;, \tag{1}$$

$$\vec{G} = x\hat{\imath} + y\hat{\jmath} \;, \tag{2}$$

$$\vec{H} = x\hat{\imath} - y\hat{\jmath} \;, \tag{3}$$

$$\vec{Q} = y\hat{\imath} - x^2y\hat{\jmath} + x\hat{k} \;, \tag{4}$$

4. Consider the vector field $\vec{F} = 5y\hat{\jmath}$.

   (a) Use the geometric definition to find the divergence at $(0, 0, 0)$. Use a cube of side c centered at $(0, 0, 0)$.

   (b) Use the geometric definition to find the divergence at $(3, 3, 0)$. Use a cube of side c centered at $(3, 3, 0)$.

   (c) Compute the divergence of $\vec{F}$ and confirm that it is consistent with your answers above.