20.2: The Divergence Theorem

Calculus III

College of the Atlantic

1. Find the divergence of the following vector fields:

$$\vec{F} = 3\hat{i} + 4\hat{j} \,, \tag{1}$$

$$\vec{F} = y\hat{i} - x\hat{j} \,, \tag{2}$$

$$\vec{F} = y^2\hat{i} - x^2\hat{j} \,, \tag{3}$$

$$\vec{F} = x\hat{i} + y\hat{j} \,, \tag{4}$$

$$\vec{F} = x\hat{i} - y\hat{j} \,, \tag{5}$$

2. Consider the vector field $\vec{F} = z^2\hat{k}$. Calculate the total flux out of a cube of side c, centered at the point $(1, 2, 3)$. Do this two ways:
   (a) By evaluating the flux integrals directly.
   (b) By using the divergence theorem.

3. Consider the electric field $\vec{E} = \vec{r}/r^3$. We have seen that $\nabla \cdot \vec{E} = 0, r \neq 0$.
   (a) Directly calculate the flux flowing out of a sphere of radius a centered at the origin.
   (b) Can we use the divergence theorem to evaluate the flux integral? Why or why not?