20.3 and 20.4: Curl and Stokes' Theorem

Calculus III

College of the Atlantic

1. Let $\vec{G} = 2y\hat{i} - 2x\hat{j}$

   (a) Sketch or describe the field.

   (b) Calculate $\nabla \times \vec{G}$ and make a sketch of it.

   (c) Use Stokes' Theorem to calculate $\int_C \vec{G} \cdot d\vec{r}$ where:

      i. C is a circle parallel to the yz-plane of radius a, centered at the origin, oriented counter clockwise when viewed from the positive x axis.

      ii. C is a circle parallel to the xy-plane of radius a, centered at a point on the z-axis, oriented counter clockwise when viewed from above.

2. Use Stokes' theorem to find the circulation of $\vec{F}$ around the circle $x^2 + y^2 = 4$, $z = 1$, oriented counterclockwise when viewed from above, and where

$$\vec{F} = (z - 2y)\hat{i} + (3x - 4y)\hat{j} + (z + 3y)\hat{k} \, . \tag{1}$$

3. Use the divergence theorem to find the flux of $\vec{F}$, given above in Eq. (1), out of a sphere of radius 3 centered at the origin.

4. Let $\vec{F}(x, y, z)$ be a vector field and $f(x, y, z)$ is a scalar function of three variables. Which of the following quantities are vectors and which are scalars. Which are not defined?

   (a) $\text{div}\, \vec{F}$
   (b) $\text{curl}\, \vec{F}$
   (c) $\text{div}\, f$
   (d) $\text{curl}\, f$
   (e) ∇f
   (f) $\nabla \vec{F}$
   (g) $\nabla \times \vec{F}$
   (h) $\nabla \cdot \vec{F}$
   (i) $\nabla \cdot f$
   (j) $\nabla \times f$
   (k) $\nabla \times \nabla f$
   (l) $\nabla \cdot \nabla \times \vec{F}$

   Are any of these quantities automatically zero?