Homework Assignment Three

Chaos and Complex Systems

Due Friday October 22, 2004.

1. Determine the cardinality of the following sets. Be sure to define terms and make your argument clear. For example, if you state that a set is countably infinite, you should explain what this means.

   (a) The set consisting of all perfect cubes $(0, 1, 8, 27, \ldots)$.

   (b) All the real numbers in the interval $[0, 2]$.

2. Consider the Boolean network shown in Fig. 1.

   (a) Determine the global truth table like we did in class.

   (b) Determine the complete dynamics of the network. How many attractors are there?

   (c) Classify each attractor (fixed point, cycle of period 4, etc.)

3. This question is very closely based on question 2.5, p. 98 from Kaplan and Glass, Understanding Nonlinear Dynamics. Springer-Verlag. 1995. Consider a 1D cellular automaton where each cell takes as input its two neighboring cells, but not from itself. The rule is $00 \mapsto 0$, $01 \mapsto 1$, $10 \mapsto 1$, $11 \mapsto 0$. Assume that the width of the CA is 10 sites. Use periodic boundary conditions.

   (a) Assume that the initial condtion has three sites on (1) and the rest off (0). Choose the on sites at random and then sketch the time evolution of the CA.

   (b) Consider the long-time behavior of the system described above, but starting with any initial condition. Is there an initial condition that will give rise to:

      i. A fixed point

      ii. A cycle

      iii. Aperiodic dynamics

   (c) Give an upper limit (if one exists) for the length of a cycle in this system.

4. Find a big map of the coast of Maine. Measure the length of the coast using successively smaller "sticks" of length epsilon. Use the relation between total length and epsilon to estimate the box-counting dimension. Your estimate should also include an estimate of your error. If you want, you can work with a friend on this one and generate only one set of data. But you should do your own discussion of the data. If you have trouble with this, see the handouts from "Chaos and Fractals Simplified for the Life Sciences."

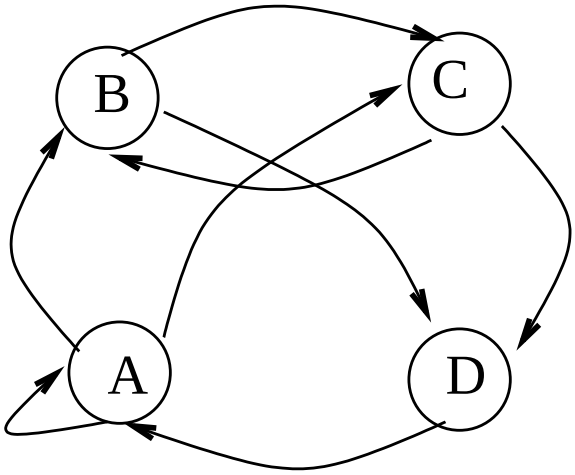

Figure 1: A Boolean network. The logic functions for each node are at the end of this document.

Truth tables for nodes of the boolean network

1	4	1
0	0	1
0	1	0
1	0	0
1	1	1

1	3	2
0	0	1
0	1	0
1	0	0
1	1	1

1	2	3
0	0	1
0	1	0
1	0	1
1	1	0

2	3	4
0	0	0
0	1	0
1	0	1
1	1	0