Homework Assignment Four

Chaos and Complex Systems

Due Wednesday October 18, 2006.

Note: Please be sure to include a list of the references you consulted and any students you worked with.

The second problem is based very closely on a problem from Daniel Kaplan and Leon Glass, *Understanding Nonlinear Dynamics,* Springer-Verlag, 1995. This is an excellent text; I used it the first time I taught this class. If you want to learn more about chaos and nonlinear dynamics, this is a great place to start. It is on reserve in the library.

1. Consider the function $f(x) = x^3$.
   (a) Plot the function. There are three fixed points. Find the fixed points.
   (b) By graphically iterating initial conditions near the fixed points, determine the stability of each.
   (c) Using the algebraic criteria (i.e. by finding the slope at the fixed points), determine the stability of all fixed points.
   (d) Does your algebra agree with the graphical results?
2. For the logistic map
   (a) Calculate the fixed point as a function of r.
   (b) Algebraically determine the r value at which the fixed point loses stability.
   (c) Does the bifurcation diagram confirm your algebraic calculation?
3. The population of a species is described by the equation $f(x) = axe^{-x}$, where $x \geq 0$ and a is a positive constant.
   (a) Determine the fixed points.
   (b) Evaluate the stability of the fixed points.
   (c) For what value of a is the first period-doubling bifurcation?
   (d) For what values of a will the population go extinct starting from any initial condition?
   (e) (optional) Using a computer, generate a bifurcation diagram as a function of a.

4. Calculate the Lyapunov exponent for the logistic equation with:

   (a) $r = 2.5$.

   (b) $r = 3.0$.

   (c) $r = 3.2$.