

Homework Assignment Six

Chaos and Complex Systems

Due Friday November 3, 2006.

1. Consider the Boolean network shown in Fig. 1.
 - (a) Determine the global truth table like we did in class.
 - (b) Determine the complete dynamics of the network. How many attractors are there?
 - (c) Classify each attractor (fixed point, cycle of period 4, etc.)
2. This question is very closely based on question 2.5, p. 98 from Kaplan and Glass, Understanding Nonlinear Dynamics. Springer-Verlag. 1995. Consider a 1D cellular automaton where each cell takes as input its two neighboring cells, but not from itself. The rule is $00 \mapsto 0$, $01 \mapsto 1$, $10 \mapsto 1$, $11 \mapsto 0$. Assume that the width of the CA is 10 sites. Use periodic boundary conditions.
 - (a) Assume that the initial condition has three sites on (1) and the rest off (0). Choose the on sites at random and then sketch the time evolution of the CA.
 - (b) Consider the long-time behavior of the system described above, but starting with any initial condition. Is there an initial condition that will give rise to:
 - i. A fixed point
 - ii. A cycle
 - iii. Aperiodic dynamics
 - (c) Give an upper limit (if one exists) for the length of a cycle in this system.

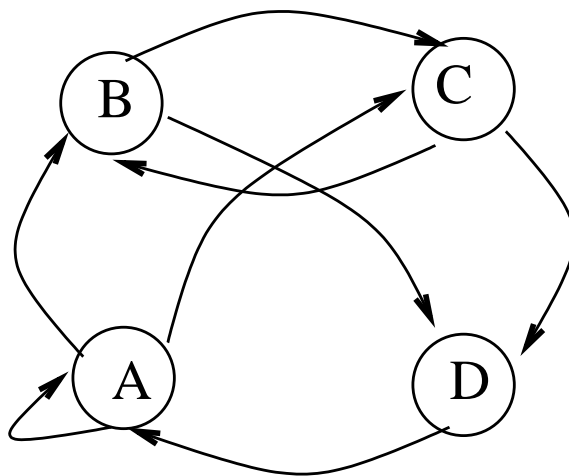


Figure 1: A Boolean network. The logic functions for each node are at the end of this document.

Truth tables for nodes of the boolean network

1	4	1
0	0	1
0	1	0
1	0	0
1	1	1

1	3	2
0	0	1
0	1	0
1	0	0
1	1	1

1	2	3
0	0	1
0	1	0
1	0	1
1	1	0

2	3	4
0	0	0
0	1	0
1	0	1
1	1	0