Chapter 2: Iterating Functions

Worksheet to accompany

David Feldman, *Chaos and Fractals: An Elementary Introduction*,
Oxford University Press, 2012

1. **Orbits of $f(x) = \frac{1}{2}x + 2$.**

   (a) Complete the first 4 iterates of the function $f(x)$ for the following seeds.

   i. $x_0 = 8$
   ii. $x_0 = 6$
   iii. $x_0 = 4$
   iv. $x_0 = 0$
   v. $x_0 = -2$

   (b) Summarize your findings. Describe the global dynamics of f. Are there any fixed points? Are there any cycles? Are the fixed points attracting or repelling?

   (c) How could you solve for any fixed points exactly, without having to carry out the iteration? Figure out a method, and implement it.

2. **Orbits of $g(x) = x^2$.**

   (a) Compute the first several iterates for several different seeds for the function $g(x)$. Choose enough seeds so you can get a sense of the overall dynamics of the function. Are there any fixed points? If so, are they attracting or repelling? In your analysis, choose only non-negative seeds.

   (b) Write down an equation that you can use to solve for the fixed point(s) of this function. Is this equation consistent with the fixed points you found above?

3. **Orbits of $h(x) = x^2 - 3$.**

   (a) Calculate the first few iterates of the following seeds for $h(x)$:

      i. $x_0 = 0$
      ii. $x_0 = 1$
      iii. $x_0 = 2$
      iv. $x_0 = 3$

   (b) How would you describe the long-term behavior of the orbit for each of these seeds?