Chapter 24: Iterating Complex Numbers

Worksheet to accompany

David Feldman, *Chaos and Fractals: An Elementary Introduction,*
Oxford University Press, 2012

Consider the squaring function $f(z) = z^2$. This is just like the squaring function we've been studying since the first day of the term, except now the inputs and outputs can be complex numbers.

1. Compute the first few iterates of the following initial conditions:

   (a) $z_0 = 1$

   (b) $z_o = i$

   (c) $z_0 = 1 + i$

   (d) $z_0 = 0.5 + 0.5i$

2. Which of the initial conditions considered above are in the Julia set[1] for $f(z) = z^2$?

3. In your opinion, bearing in mind that it is 2020, does iterating complex numbers seem like a task that should be done by machines and not humans?

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[1]Recall that a point is in the Julia set for a function if its orbit does not tend toward infinity (positive or negative). For complex numbers, "tend toward infinity" means moves further and further away from the origin. It doesn't matter in which direction it is moving away from the origin or if it is spiraling while doing so.