Some Midterm Thoughts

Chaos and Fractals

Spring 2026

College of the Atlantic.

Character of phenomena $\neq$ character of process generating phenomena:

Observed Phenomena	Cause or Explanation	Examples
Randomness, Unpredictability, Butterfly Effect	Determinism	Logistic Equation and many other systems
Complexity	Simple Rule	Large periods of logistic equation, Mandlebrot set, some CAs
Intricate Fractals	Deterministic Iterative Rule	Building Cantor Set, Sierpinski Triangle, etc., by removing pieces of original shape, cutting and folding paper, algorithmic drawing
Intricate Fractals	Stochastic (non-deterministic) Rule	The Chaos Game, flipping coins to make Sierpinski triangle
Large-scale Pattern	Local Rule	Cellular automata, strange attractors

Topics covered so far:

1. Chaos, aperiodicity, butterfly effect
2. Bifurcation diagram, universality of period doubling
3. Fractals and non-integer dimension
4. Local instability (weather) and global stability (climate)
5. Strange attractors combine order and disorder
6. Power laws and fractal distributions. Averages do not always exist (St. Petersburg game)
7. Cellular automata
8. Different types of infinity

Topics to be covered in the next few weeks:

0. Complex numbers (today!)
1. Julia and Mandelbrot sets
2. Strange attractors
3. Some philosophy of science
4. Hitomezashi patterns
5. Turing patterns and reaction–diffusion (maybe)

Some themes:

1. Dynamical systems combine order and disorder: deterministic chaos, butterfly effect, statistical stability of chaotic orbits.
2. There are many *very* different ways to generate fractals.
3. Concepts and ideas which I hope are seeming more complicated, interesting, and richer than at the start of the course: **Simple, Complex, Random, Deterministic, Order, Disorder, Infinity**.