Some Parting Thoughts

Chaos and Fractals

Winter 2022

College of the Atlantic.

The axioms[1] for this course were:

1. Mathematical potential is distributed equally among different groups, irrespective of geographic, demographic, and economic boundaries.
2. Everyone can have joyful, meaningful, and empowering mathematical experiences.
3. Mathematics is a powerful, malleable tool that can be shaped and used differently by various communities to serve their needs.
4. Every student deserves to be treated with dignity and respect.

The community agreement[2] for this course was:

> This course aims to offer a joyful, meaningful, and empowering experience to every participant; we will build that rich experience together by devoting our strongest available effort to the class. You will be challenged and supported. Please be prepared to take an active, critical, patient, creative, and generous role in your own learning and that of your classmates.

The goals for this course were:

1. Stay physically and mentally healthy and maintain intellectual and personal connection during a potentially difficult time.
2. I want you to gain a solid understanding of the basic mathematical ideas behind chaos and fractals. As part of this, I hope you'll leave this course thinking somewhat differently about order and disorder, simplicity and complexity.
3. I want to help you improve your basic facility with algebra and functions, your problem solving skills, your ability to create and interpret different types of graphs, and your overall mathematical confidence.
4. I want you to gain a sense of how math and physics is done, and gain an awareness that these are not static, "dead" disciplines. I want you to gain a greater understanding of science as an institution and science as a culture, and how science intersects and influences other creative and scholarly areas.
5. Have fun while growing and learning.

[1] These axioms were written by Federico Ardila-Mantilla.

[2] Based on Ardila-Mantilla's community agreement for his classes.

Character of phenomena $\neq$ character of process generating phenomena:

Observed Phenomena	Cause or Explanation	Examples
Randomness, Unpredictability, Butterfly Effect	Determinism	Logistic Equation and many many many any other things
Complexity	Simple Rule	Large periods of logistic equation, Mandlebrot set, some CAs
Intricate Fractals	Precise, Deterministic Rule	Building Cantor Set, Sierpinski Triangle, etc., by removing pieces of original shape, cutting and folding paper
Intricate Fractals	Stochastic (i.e., non-deterministic) Rule	The Chaos Game, flipping coins to make Sierpinski triangle
Large-scale Pattern	Local Rule	Cellular automata, strange attractors

Some key material:

1. Chaos, aperiodicity, butterfly effect
2. Bifurcation diagram, universality of period doubling
3. Fractals and non-integer dimension
4. Local instability (weather) and global stability (climate)
5. Strange attractors combine order and disorder
6. Power laws and long tails. Averages do not always exist (St. Petersburg game)
7. Cellular automata
8. Different types of infinity
9. Julia and Mandelbrot sets
10. Uses of mathematical models
11. Sexism, misogyny, and racism in math and physics

Concepts and ideas which I hope seem more complicated, interesting, and richer than at the start of the course: **Simple, Complex, Random, Deterministic, Order, Disorder, Infinity**.