Topology and Möbius Strip Explorations

Chaos and Fractals

College of the Atlantic.

This lab is only indirectly related to chaos. It's designed to give you a small taste of the mathematical field known as topology, which is mentioned in Gleick's book. Topology is the study of properties shapes that remain unchanged when the shapes are stretched. Topology is concerned with how shapes are connected—how many edges and sides they have, for example—and not with the exact form or size of an object. (Mostly, though, the goal of these explorations is to have fun and stretch your mind.)

Do these exercises in groups of two or three. Check with me as you do these if you have any questions or if the instructions aren't clear.

1. Loops and Twists:

   (a) Make a loop out of a piece of paper. How many edges does it have? How many sides? How can you tell?

   (b) Make a Möbius strip by giving the paper a half twist before taping it together. How many sides does it have? How many edges? In answering these questions, it will help to draw a line around one loop of the paper.

   (c) Make another loop, this time by giving the paper a full twist (i.e., two half twists) before taping it together. How many sides does it have? How many edges?

2. Dissecting:

   (a) What will happen to a Möbius strip if you cut it in half lengthwise? Ponder, debate, and make predictions.

   (b) Cut it in half and see. Was your prediction right? How would you explain the result? How many edges and sides do you now have? How many twists are there? Why?

   (c) Start with a new Möbius strip. Cut it in thirds. To do so, start cutting about a third of the way in from one of the edges. Then just cut around lengthwise. You'll end up going around the loop twice. What do you think will happen when you do this?

   (d) Try it and see. What just happened? Why? Try the above experiment again with a fresh Möbius strip. Before you trisect it, use a marker to color the entire edge of the strip. Then write A on the left third, B on the center third, and C on the right third. I'll show you what this looks like if it's not clear. Then trisect again and the markings you made will likely help you see how the impossible thing that happened is possible.

3. Pairs of loops:

   (a) Take two regular loops and tape them together at right angles. Dissect both loops. What happens? Why?

   (b) Take one regular loop and one Möbius strip and tape them together at right angles. Dissect both loops. What happens? Why?

   (c) Take two Möbius strips and tape them together at right angles. Important: The möbius strips need to have opposite twists. Turn one a half-turn clockwise and the other a half-turn anti-clockwise. Dissect both loops. What happens?

   (d) Take two Möbius strips and tape them together at right angles. This time make möbius strips twisted in the same way. Dissect both loops. What happens? The situation started off symmetric. Does it end up symmetric?