Dynamical Systems
Homework Eleven
Due February $\heartsuit + 4$, 2013

Figure 1: A Dilbert Cartoon. Scott Adams. © Universal Uclick.

Central Limit Theorem

1. Write a function that returns 1 with probability 0.5, 2 with probability 0.1, 4 with probability 0.1 and 6 with probability 0.3. Let's refer to the value that is returned as X.

2. Call this function many times, storing the X values in a list. Let M be the number of times the function is called. You will experiment with different M values, so write your code so that it is easy to change.

3. Plot a histogram of the X values. Does it look like you'd expect?

4. Now imagine calling this function twice and then averaging the two X values. Call this average S_2. I.e.,
$$S_2 = \frac{X_1 + X_2}{2} . \tag{1}$$
Repeat this M times, storing the S_2 values in a list. Make a histogram for this data.

5. Repeat the above, but for sums of larger numbers of X's. Let
$$S_n = \frac{1}{n} \sum_i n X_n . \tag{2}$$
Calculate histograms for, say, S_3, S_5, and S_{10}. What do the histograms look like?

Figure 2: Michele C. Hénon. Figure source: `www.espace-turing.fr/-ENI-Experimentation-Numerique-.html`.

The Hénon Map

The Hénon map is a two-dimensional, discrete dynamical system defined by:

$$x_{n+1} = y_n + 1 - ax_n^2 \,, \tag{3}$$

$$y_{n+1} = bx_n \,, \tag{4}$$

where a and b are parameters. Write some python code to explore the map. Since there are two variables and two parameters, there are a number of different ways to view the dynamics. Try making separate time series plots, as we've done for two- and three-dimensional differential equations. Also make some phase-space plots: plot x and y against each other, leaving out time.

Experiment with different parameter values. Try making some one- and two-dimensional bifurcation diagrams.