Dynamical Systems
Homework Two
Due January 14, 2013

Figure 1: A cup of coffee. Figure source: George Shulkin. `http://en.wikipedia.org/wiki/` `File:A_small_cup_of_coffee.JPG`

.

Please do these before class on Monday. We will discuss these exercises in class.

1. Consider the differential equation

$$\frac{dT}{dt} = -0.2(T(t) - 20) , \tag{1}$$

   where $T(0) = 80$. Such an equation might describe the temperature T of a cup of coffee that is at 80 at $t = 0$ when it is placed in a 20 degree room.

   (a) Use Euler's method with $\Delta T = 2$ to come up with an estimate for $T(10)$.

   (b) Verify by substitution that

   $$T(t) = 20 + 60e^{-0.2t} , \tag{2}$$

   is a solution to Eq. (1).

2. Consider the following differential equation:

$$P' = \frac{1}{500}P(P - 50)(200 - P) , \quad \text{for } P \geq 0 . \tag{3}$$

   (a) Sketch a plot of the right-hand side of Eq. (3).

   (b) Use your plot to sketch a slope field for Eq. (3).

   (c) Find and classify all equilibrium points.

   (d) To what sort of phenomena might this model apply?

3. Optional programming challenge. Do this if it seems fun or interesting. Write some python code that implements Euler's method for a differential equation. You can test it on the cooling coffee example above. Have your code plot the Euler solution.