Dynamical Systems
Homework Four
Due January 24, 2013

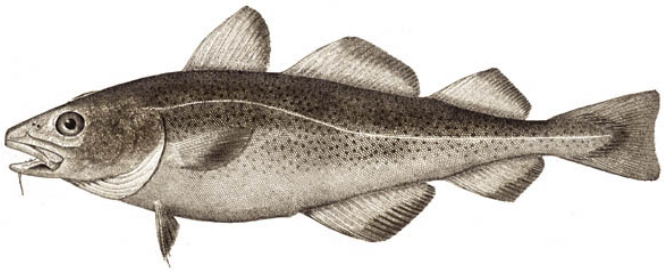

Figure 1: Atlantic cod. Figure source: `http://en.wikipedia.org/wiki/File:Atlantic_cod.jpg`

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1. Consider the function $f(x) = \sin(\pi x)$. Modify your bifurcation diagram program to plot the bifurcation diagram to iterates of this function. Note that you will need to import `sin` from `math`.

2. Back to ordinary differential equations. We will carry out a qualitative analysis of the logistic differential equation with harvesting:

$$\frac{dP}{dt} = \frac{1}{20}P(1 - \frac{P}{500}) - h\,. \tag{1}$$

   In the above equation, h is the rate at which the population is harvested, and we are only interested in positive populations. We will see what happens as we vary h.

   (a) For each of following values of h, sketch the right-hand side of Eq. (1), draw the phase line, and sketch solutions to the differential equation for a few different initial populations.

      i. $h = 0$
      ii. $h = 3$
      iii. $h = 6$
      iv. $h = 9$

   (b) Use your phase line sketches to produce a bifurcation diagram for Eq. (1). (We haven't done this yet for an ODE. So if it's not clear what I'm asking, ponder it for a moment but don't worry. We'll go over this in class.)