Dynamical Systems
Homework Six
Due January 28, 2013

$$\varepsilon$$

Figure 1: An ϵ. Figure source: `http://en.wikipedia.org/wiki/File:Greek_lc_epsilon.svg`.

A function on the unit interval has sensitive dependence on initial conditions if for any δ, x_1, and $\epsilon \in [0, 1]$, there exists a natural number n and an $x_2 \in [0, 1]$ such that $|x_1 - x_2| < \epsilon$ and $|f^{(n)}(x_1) - f^{(n)}(x_2)| < \delta$.

1. Let $\delta = 0.7$, $x_1 = 0.4$ and $\epsilon = 0.05$. Find an n and an x_x such that $|x_1 - x_2| < \epsilon$ and $|f^{(n)}(x_1) - f^{(n)}(x_2)| < \delta$. (This is not a deep problem. Just use one of your programs to experiment with different x_2's.) Make a plot of the two orbits that demonstrates that they meet the criteria.

2. Repeat the above exercise, but let $\epsilon = 0.01$.

3. Consider the differential equation

$$\frac{dx}{dt} = x^3 - ax. \tag{1}$$

   (a) For each of the cases below, do the following. Sketch the right hand side of Eq. (1). Sketch the phase line. How many equilibria does the equation have? Are they stable or unstable?

      i. $a < 0$
      ii. $a = 0$
      iii. $a > 0$

   (b) Use your phase line sketches to produce a bifurcation diagram for Eq. (1).