

**Dynamical Systems**  
**Homework Seven**  
**Due February 4, 2013**



Figure 1: A Pitchfork. Figure source: <http://www.amazon.com/Tine-Pitchfork-Non-Splinter-Fiberglass-Shaft/dp/B001AZL0EA>.

1. Suppose a dynamical system has a Lyapunov exponent of 0.7. Two initial conditions are 0.05 apart. Approximately how far apart would you expect them to be after two iterations? How far apart would you expect them to be after six iterations?
2. Suppose a dynamical system has a Lyapunov exponent of  $-0.5$ . Two initial conditions are 0.1 apart. Approximately how far apart would you expect them to be after two iterations? How far apart would you expect them to be after six iterations?
3. Determine the bifurcation diagram for

$$\frac{dx}{dt} = rx + x^3 - x^5. \quad (1)$$

This bifurcation diagram is a good bit more complex (and more interesting) than the ones we have encountered so far. You will need to try out a number of different  $r$  values to get the full diagram. Optional: If you feel like practicing some calculus, determine the  $r$  and  $x$  values for all bifurcations.