Transformations

1. Determine the single matrix that performs the following operations:
   (a) A rotation of $\pi/3$ followed by a dilation of 3.
   (b) A rotation of $\pi/3$ followed by a dilation of 3.
   (c) A dilation of 3 followed by a rotation of $\pi/3$.

2. Determine whether or not the following transformations are linear:
   (a) $T : \mathbb{R}^3 \to \mathbb{R}^3$, $T(x, y, z) = (2x, 3z, x + z)$.
   (b) $T : \mathbb{R}^3 \to \mathbb{R}^2$, $T(x, y, z) = (2x, yz)$.
   (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$, $T(x, y) = (2y, 2 + 4)$.
   (d) $\frac{d}{dx} : f(x) \mapsto f'(x)$

3. Suppose people either buy or rent a house. Every year, 15% of homeowners sell their houses and rent, while 10% of renters buy a house. Set up a transition matrix for this situation. If initially there are 200 owners and 400 renters, what will the situation be in two years?