Vector Spaces

Which of the following sets are vector spaces. Unless otherwise stated, assume that "+" and "·" have their usual meaning.

1. The vector $\vec{0} = (0, 0, 0, 0)$

2. The vectors $\vec{v_1} = (1, 0)$ and $\vec{v_2} = (2, 3)$

3. $V = \{\, \vec{v} = (4 - 3r, 1 - 2r, r) \,|\, r \in \mathbb{R} \,\}$

4. $W = \{\, \vec{w} = (a^2, a, b) \,|\, a, b \in \mathbb{R} \,\}$

5. $U = \{\, \vec{u} = (u_1, u_2, u_3) \,|\, u_i \in \{0, 1\} \,\}$

6. $B = \{\, f(x) = b_0 + b_1 x + b_2 x^2 \,|\, b_i \in \mathbb{R} \,\}$

7. The set $\{\, f \,|\, f : \mathbb{N} \to \mathbb{R} \,\}$ of all real-valued functions of one natural number variable, where the addition and scalar multiplication operations are defined by:

$$(f_1 + f_2)(n) = f_1(n) + f_2(n) \,, \tag{1}$$

and

$$(r \cdot f)(n) = r f(n) \,. \tag{2}$$