

Linear Algebra Homework Seven

Due Friday November 5, 2010

For the sage portions of this assignment, please hand in hard copy of your sage worksheet.

1. **Sage and Numbers and Algebra and Calculus.** Have sage do the following.

(a) Evaluate the following. Have sage give both an exact and a decimal answer, if possible.

i. $\log(3^9)$

ii. $\frac{2}{3} + \frac{3}{5} + \frac{5}{7}$

iii. $\sin(\pi/2)$

(b) Use sage to solve the following equations:

i. $3x - 8 = 4$

ii. $4x^2 + 8x = 27$

iii. $50 = 100(0.88)^x$

iv. $\cos(x) = x$

(c) Use sage to evaluate the following derivatives:

i. $f(x) = x^{13}$

ii. $f(x) = \cos(2x^3 - 4x)$

iii. $f(x) = \ln(4x^2)$

iv. $f(x) = e^{-x^2}$

(d) Use sage to evaluate the following definite and indefinite integrals:

i. $\int x^{13} dx$

ii. $\int \cos(x) \sin(x) dx$

iii. $\int_0^7 3x^4 dx$

iv. $\int_1^4 x \cos(x^2) dx$

2. **Sage and Linear Algebra:** Consider the following matrix:

$$A = \begin{pmatrix} 9 & 3 & 2 & 0 \\ 2 & 0 & -1 & 3 \\ -1 & 4 & 2 & 0 \\ 2 & 3 & 8 & 3 \end{pmatrix} \quad (1)$$

Calculate the following quantities:

(a) A^T

(b) A^{-1}

(c) A^3

(d) A^{10}

3. **Sage and a Markov Chain:** Suppose the weather can be either R (rainy), C (cloudy) or S (sunny). Let the transition matrix be given by:

$$C = \begin{pmatrix} 0.80 & 0.70 & 0.50 \\ 0.15 & 0.20 & 0.30 \\ 0.05 & 0.10 & 0.20 \end{pmatrix} \quad (2)$$

- (a) Suppose it is rainy today. What is the probability of the three weather types in one day? In 10 days? In 100 days?
- (b) Suppose it is sunny today. What is the probability of the three weather types in one day? In 10 days? In 100 days?
- (c) Suppose it is cloudy today. What is the probability of the three weather types in one day? In 10 days? In 100 days?
- (d) What do you conclude about the frequency, in the long run, of rainy, cloudy, and sunny days?
4. **Sage and a Structured Population Model:** Suppose giraffes can be viewed as belonging to one of four age classes. There is a 60 percent probability of transitioning from age class 1 to 2, 2 to 3, and 3 to 4. If a giraffe does not transition, it dies. Once in age class four, it remains in age class four with a probability of 80 percent. Giraffes in age class 2 have a seventy percent chance of having a baby and giraffes in age class 3 have a 60 percent chance of having a baby.
- (a) Suppose there are initially 100 giraffes in age class 1. What is the population distribution next year? In 10 years? In 100 years?
- (b) Is the long-term behavior of the population stable, or do the giraffes all die, or do the giraffes take over the world?
- (c) Try modifying the model to produce the opposite behavior. I.e., if the giraffes all die in the long run, make some modifications that will make the giraffes take over the world in the long run.

5. **Chapter 3.1:**

- (a) 2
(b) 6
(c) 13
(d) 18

6. **Chapter 3.2:**

- (a) 1
(b) 5
(c) 6
(d) 9
(e) 19