

Linear Algebra

Exercises for Lecture One

Due Tuesday, September 17, 2013 (?)

1. Determine the value of Ax , where

$$A = \begin{pmatrix} 1 & -2 \\ 2 & 4 \end{pmatrix} \tag{1}$$

$$x = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{2}$$

Carry out this multiplication two ways:

- (a) By forming a linear combination of the columns of A .
- (b) By taking two dot products (aka “across-by-down”).

2. Consider the following system of equations:

$$2x + y = 3, \tag{3}$$

$$x - 2y = -1. \tag{4}$$

- (a) Solve the equations using the row picture. Sketch the two lines and determine their point of intersection.
- (b) Solve the equations using the column picture. Sketch the two columns and illustrate geometrically the linear combination of the two columns that yields $(3, -1)$.

3. Consider the system $Ax = b$, where

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & -1 \\ 1 & 0 & 0 \end{pmatrix} \tag{5}$$

and

$$b = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{6}$$

- (a) Determine the solution to the system. Use the column picture.
- (b) Can you solve $Ax = b$ for any b ? It may help to sketch the three columns.
- (c) Do all linear combinations of the columns of A fill 3D space? If not, what do all the linear combinations look like?

4. Repeat the above question, but let

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad (7)$$

and

$$b = \begin{pmatrix} 6 \\ 2 \\ 2 \end{pmatrix} \quad (8)$$