

Linear Algebra

Exercises for Lecture Twenty-Two: Markov Matrices

Due Tuesday, November 12, 2013

- Chapter 8.3, problem 1
- Chapter 8.3, problem 5
- In a certain town the weather can either be rainy or sunny. Data taken over a number of years has shown that if it is rainy on one day, there is a 60% chance that it will be rainy the next day. And if it is sunny on one day, there is a 70% chance it will be rainy the next day.
 - Construct a Markov transition matrix that describes this situation. Call this matrix A .
 - What is the meaning of the elements of the matrix A^9 ?
 - It is rainy today. What is the probability that it is rainy tomorrow?
 - It is rainy today. What is the probability that it is rainy 7 days later?
 - In the long run, what fraction of the days are rainy?
 - What is A^k in the limit that k goes to infinity?
- Consider the two vectors

$$q_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad q_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (1)$$

- Verify that these vectors are orthonormal.
- We can write any vector v as a linear combination of the q 's:

$$v = c_1 q_1 + c_2 q_2. \quad (2)$$

Write down a general formula for c_1 and c_2 .

- Use the formula you just wrote down to solve for c_1 and c_2 for the vector $v = (1 \ 4)$.
- Make a sketch of the situation and show the geometric meaning of c_1 and c_2 .