

## Linear Algebra

### Exercises for Lecture Nine: Independence, Basis, and Dimension

Due Friday, October 11, 2013

1. Consider the following three vectors:

$$v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (1)$$

$$v_2 = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (2)$$

$$v_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (3)$$

- (a) What is the span of the three vectors?
- (b) What is the dimension of the space that they span?
- (c) Do the three vectors span  $\mathbb{R}^3$ ?
- (d) Are the three vectors independent?
- (e) If possible, find non-zero  $c_1, c_2, c_3$  such that:

$$c_1v_1 + c_2v_2 + c_3v_3 = 0. \quad (4)$$

- (f) Write down a basis for the space spanned by  $v_1, v_2$ , and  $v_3$ .
- (g) Write down another basis for the space spanned by  $v_1, v_2$ , and  $v_3$ .

2. Consider the matrix  $A$ :

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 0 & 4 & 12 \end{pmatrix} \quad (5)$$

- (a) Do the columns of  $A$  span  $C(A)$ ?
- (b) Are the columns of  $A$  independent? Answer this question by determining  $\text{ref}(A)$ .
- (c) What is the dimension of  $C(A)$ ?
- (d) What is  $N(A)$ ?
- (e) What is the dimension of  $N(A)$ ?

- 3. Chapter 3.5, problem 2
- 4. Chapter 3.5, problem 5
- 5. Chapter 3.5, problem 15
- 6. Chapter 3.5, problem 18