Linear Algebra

Exercises for Population Models

Due Friday, November 15, 2013

These questions are based on exercise 2.13 from Ellner and Guckenheimer, *Dynamic Models in Biology*, Princeton University Press, 2006. The data is from Brault and Caswell, Pod-specific demography of killer whales (*Orcinus ocra*). Ecology 74:1444–1454, 1993.

Consider a four-stage population model whose transitions are given by the following projection matrix:

$$A = \begin{pmatrix} 0 & 0.0043 & 0.1132 & 0 \\ 0.9775 & 0.9111 & 0 & 0 \\ 0 & 0.0736 & 0.9534 & 0 \\ 0 & 0 & 0.0452 & 0.9804 \end{pmatrix} \tag{1}$$

You will want to use a computer to answer these questions. There are lots of choices: Wolframalpha, R, python, maple, matlab, etc.

1. Determine the dominant eigenvalue λ and its eigenvector u.

2. What is the biological significance of λ?

3. What is the biological significance of u?

4. Suppose that population this year is $u_0 = (10, 60, 110, 70)$. What is the population next year? What is the population two years from now?

5. Optional: Requires a bit more computer work. Using the initial population given above, write a program that produces on there separate graphs the following:

   (a) $N(t)$, the total population in year t
   (b) The annual population growth rate $N(t + 1)/N(t)$.
   (c) The proportion of individuals in each of the four stages

   Discuss your results and compare with your answers to questions 2 and 3.