Linear Algebra Homework Ten

College of the Atlantic

Due Friday March 15, 2019

Please include a cover sheet for this assignment.

Chapter 7.5

1. 3
2. 7

Linearization and Jacobians

1. Consider the nonlinear system:
$$x' = -2x + 2x^2 \, , \tag{1}$$
$$y' = -3x + y + 3x^2 \, . \tag{2}$$

   (a) Find all equilibria for this system. Hint: there are two.

   (b) Determine the Jacobian matrix.

   (c) Use the Jacobian matrix to classify all equilibria.

2. The *Lotka–Volterra* equations are the simplest model of a predator–prey interaction. The equations are:
$$x(t)' = Ax - Bxy \, , \tag{3}$$
$$y(t)' = Cxy - Dy \, , \tag{4}$$

   where A, B, C, and D are positive constants, and $x(t)$ and $y(t)$ are the populations (or total biomass) of two different species.

   (a) Which is the population of the predators, x or y? Briefly explain.

   (b) Find all non-negative equilibria for the system. (There are two, one of which is $(0,0)$.)

   (c) Determine the Jacobian for the system.

   (d) Use the Jacobian to say as much as you can about the nature of the fixed points.

   (e) Optional, but recommended. Find the eigenvectors for the linearized system (i.e., the Jacobian) at $(0,0)$. Biologically, what do each of these eigenvectors represent?