Chapter 3.3

Linear Algebra with applications to differential equations

College of the Atlantic. Winter 2019

When taking an augmented matrix and converting it to echelon form, a number of things can happen. Here are some possibilities

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & 1 & 5 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} 1 & 2 & 1 & 4 & 0 \\ 0 & 1 & 2 & 4 & 10 \\ 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

1. (Re)introduce yourself to others in your group. Briefly share with your group-mates the names of any pets you or your family have.
2. Which of these systems has a unique solution? Which have an infinite number of solutions? Which have no solutions? Are there any free variables in any of the systems?
3. Determine the solution(s) for system (1).
4. Convert the augmented matrix in (1) into reduced row echelon form. What do you notice?
5. Determine the solution(s) for system (2).

. . . .

6. Write each of the following homogeneous systems in matrix form, use row reduction to convert the matrix to reduced echelon form, and then determine all solutions to the system.

$$\begin{aligned} 1x_1 + 2x_2 + 1x_3 &= 0 \\ 3x_1 + 8x_2 + 7x_3 &= 0 \\ 2x_1 + 7x_2 + 9x_3 &= 0 \end{aligned} \tag{6}$$

$$\begin{aligned} 1x_1 + 2x_2 + 3x_3 &= 0 \\ 3x_1 + 4x_2 + 5x_3 &= 0 \\ 4x_1 + 6x_2 + 8x_3 &= 0 \end{aligned} \tag{7}$$

. . . .

7. Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} \tag{9}$$

   (a) Calculate AB.
   (b) Calculate BA.
   (c) Calculate A^2.

. . . .

8. Consider the following three matrices

$$A = \begin{bmatrix} 4 & 1 & -2 & 7 \\ 3 & 1 & -1 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 5 \\ 3 & -1 \\ -2 & 4 \\ 2 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ -2 & 3 \\ 1 & -3 \end{bmatrix}. \tag{10}$$

   (a) Calculate AB and AC. You should find that $AB = AC$.
   (b) Does $B = C$?
   (c) That's weird, right?