Chapter 4.4 & 4.5

Linear Algebra with applications to differential equations

College of the Atlantic. Winter 2019

1. (Re)introduce yourself to your partners and briefly share something—big or small—that has pleasantly surprised you lately.

2. Write $\vec{x} = (2, -2)$ as a linear combination of $\vec{v_1} = (1, 0)$ and $\vec{v_2} = (0, 1)$.

   (a) Is the linear combination you found unique?
   (b) Are there any vectors $\vec{x}$ that cannot be written as a linear combination of $\vec{v_1}$ and $\vec{v_2}$?
   (c) Do $\vec{v_2}$ and $\vec{v_2}$. span $\mathbb{R}^2$?
   (d) Is the set consisting of $\vec{v_1}$ and $\vec{v_2}$ a basis for $\mathbb{R}^2$?

3. Write $\vec{x} = (2, -2)$ as a linear combination of $\vec{v_1} = (1, 1)$ and $\vec{v_2} = (1, -1)$.

   (a) Is the linear combination you found unique?
   (b) Are there any vectors $\vec{x}$ that cannot be written as a linear combination of $\vec{v_1}$ and $\vec{v_2}$?
   (c) Do $\vec{v_2}$ and $\vec{v_2}$. span $\mathbb{R}^2$?
   (d) Is the set consisting of $\vec{v_1}$ and $\vec{v_2}$ a basis for $\mathbb{R}^2$?

4. Write $\vec{x} = (2, -2)$ as a linear combination of $\vec{v_1} = (1, 1)$, $\vec{v_2} = (1, -1)$, and $\vec{v_3} = (1, 0)$.

   (a) Is the linear combination you found unique?
   (b) Are there any vectors $\vec{x}$ that cannot be written as a linear combination of $\vec{v_1}$, $\vec{v_2}$, and $\vec{v_3}$?
   (c) Do $\vec{v_1}$, $\vec{v_2}$, and $\vec{v_3}$ span $\mathbb{R}^2$?
   (d) Is the set consisting of $\vec{v_1}$, $\vec{v_2}$, and $\vec{v_3}$ a basis for $\mathbb{R}^2$?

5. Write $\vec{x} = (2, -2)$ as a linear combination of $\vec{v_1} = (1, -1)$.

   (a) Is the linear combination you found unique?
   (b) Are there any vectors $\vec{x}$ that cannot be written as a linear combination of $\vec{v_1}$?
   (c) Does $\vec{v_1}$ span $\mathbb{R}^2$?
   (d) Is $\vec{v_1}$ a basis for $\mathbb{R}^2$?

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6. Determine if each of the following sets of vectors are a basis for $\mathbb{R}^3$.

   (a) $\vec{v_1} = (1, 2, 0)$, $\vec{v_2} = (1, 2, -1)$, $\vec{v_3} = (1, 0, 2)$.
   (b) $\vec{v_1} = (1, 2, 2)$, $\vec{v_2} = (2, 3, 4)$, $\vec{v_3} = (3, 8, 7)$, $\vec{v_4} = (3, -8, 7)$
   (c) $\vec{v_1} = (1, 4, 0)$, $\vec{v_2} = (1, 2, -1)$.

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7. Find a basis for the row space and a basis for the column space of the matrix A:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & -9 \\ 2 & 5 & 2 \end{bmatrix}, \tag{1}$$