Activity 1.2.3 Augmented matrices and solution spaces.

a. Write the augmented matrix for the linear system

$$\begin{aligned} x + 2y - z &= 1 \\ 3x + 2y + 2z &= 7 \\ -x \qquad + 4z &= -3 \end{aligned}$$

and perform Gaussian elimination to describe the solution space in as much detail as you can.

b. Suppose that you have a linear system in the variables x and y whose augmented matrix is row equivalent to

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

Write the linear system corresponding to this augmented matrix and describe its solution set in as much detail as you can.

c. Suppose that you have a linear system in the variables x and y whose augmented matrix is row equivalent to

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right].$$

Write the linear system corresponding to this augmented matrix and describe its solution set in as much detail as you can.

d. Suppose that the augmented matrix of a linear system has the following shape where $*$ could be any real number.

$$\left[\begin{array}{ccccc|c} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{array}\right].$$

1. How many equations are there in this system and how many variables?
2. Based on our earlier discussion in Section 1.1, do you think it's possible that this system has exactly one solution, infinitely many solutions, or no solutions?
3. Suppose that this augmented matrix is row equivalent to

$$\left[\begin{array}{ccccc|c} 1 & 2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 2 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Make a choice for the names of the variables and write the corresponding linear system. Does the system have exactly one solution, infinitely many solutions, or no solutions?