Activity 1.4.2

a. Shown below are three augmented matrices in reduced row echelon form.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

For each matrix, identify the pivot positions and determine if the corresponding linear system is consistent. Explain how the location of the pivots determines whether the system is consistent or inconsistent.

b. Each of the augmented matrices above has a row in which each entry is zero. What, if anything, does the presence of such a row tell us about the consistency of the corresponding linear system?

c. Give an example of a 3×5 augmented matrix in reduced row echelon form that represents a consistent system. Indicate the pivot positions in your matrix and explain why these pivot positions guarantee a consistent system.

d. Give an example of a 3×5 augmented matrix in reduced row echelon form that represents an inconsistent system. Indicate the pivot positions in your matrix and explain why these pivot positions guarantee an inconsistent system.

e. Write the reduced row echelon form of the coefficient matrix of the corresponding linear system in Item d? (Remember that the Augmentation Principle says that the reduced row echelon form of the coefficient matrix simply consists of the first four columns of the augmented matrix.) What do you notice about the pivot positions in this coefficient matrix?

f. Suppose we have a linear system for which the *coefficient* matrix has the following reduced row echelon form.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -3 \end{bmatrix}$$

What can you say about the consistency of the linear system?

Activity 1.4.3

a. Here are the three augmented matrices in reduced row echelon form that we considered in the previous section.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

For each matrix, identify the pivot positions and state whether the corresponding linear system is consistent. If the system is consistent, explain whether the solution is unique or whether there are infinitely many solutions.

b. If possible, give an example of a 3×5 augmented matrix that corresponds to a linear system having a unique solution. If it is not possible, explain why.

c. If possible, give an example of a 5×3 augmented matrix that corresponds to a linear system having a unique solution. If it is not possible, explain why.

d. What condition on the pivot positions guarantees that a linear system has a unique solution?

e. If a linear system has a unique solution, what can we say about the relationship between the number of equations and the number of variables?