2 Vectors, matrices, and linear combinations

2.1 Vectors and linear combinations

Preview Activity 2.1.1 Scalar Multiplication and Vector Addition. Suppose that

$$\mathbf{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

a. Find expressions for the vectors

$$\begin{array}{cccc} \mathbf{v}, & 2\mathbf{v}, & -\mathbf{v}, & -2\mathbf{v}, \\ \mathbf{w}, & 2\mathbf{w}, & -\mathbf{w}, & -2\mathbf{w}. \end{array}$$

and sketch them using Figure 2.1.2.

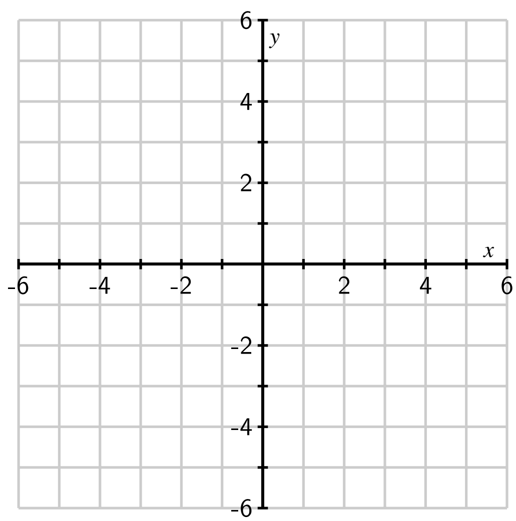

Figure 2.1.2 Sketch the vectors on this grid.

b. What geometric effect does scalar multiplication have on a vector? Also, describe the effect that multiplying by a negative scalar has.

c. Sketch the vectors $\mathbf{v}, \mathbf{w}, \mathbf{v} + \mathbf{w}$ using Figure 2.1.3.

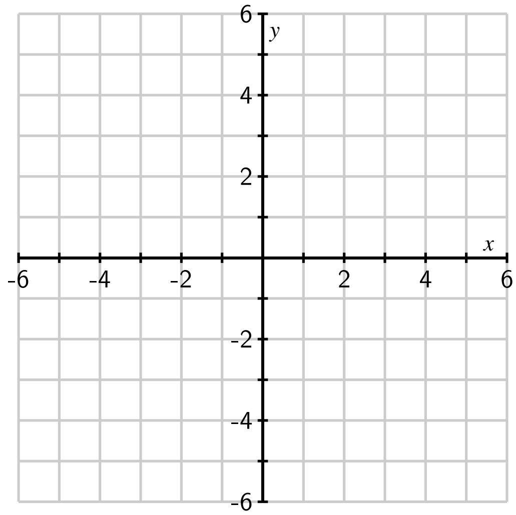

Figure 2.1.3 Sketch the vectors on this grid.

d. Consider vectors that have the form $\mathbf{v} + c\mathbf{w}$ where c is any scalar. Sketch a few of these vectors when, say, $c = -2, -1, 0, 1,$ and 2. Give a geometric description of this set of vectors.

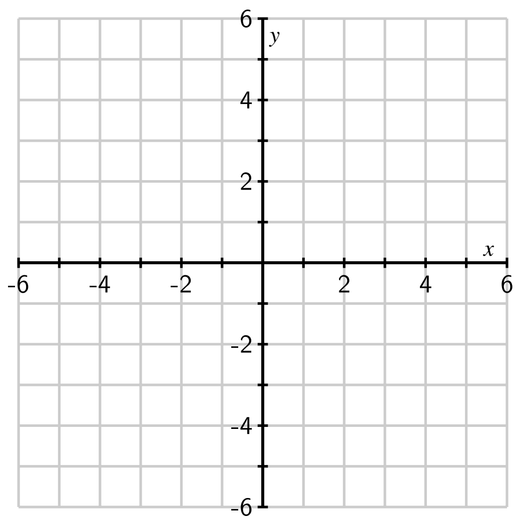

Figure 2.1.4 Sketch the vectors on this grid.

e. If c and d are two scalars, then the vector

$$c\mathbf{v} + d\mathbf{w}$$

is called a *linear combination* of the vectors **v** and **w**. Find the vector that is the linear combination when $c = -2$ and $d = 1$.

f. Can the vector $\begin{bmatrix} -31 \\ 37 \end{bmatrix}$ be represented as a linear combination of **v** and **w**? Asked differently, can we find scalars c and d such that $c\mathbf{v} + d\mathbf{w} = \begin{bmatrix} -31 \\ 37 \end{bmatrix}$.