Activity 2.2.2 Matrix-vector multiplication.

a. Find the matrix product

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 4 & -3 & -2 \\ -1 & -2 & 6 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}.$$

b. Suppose that A is the matrix

$$\begin{bmatrix} 3 & -1 & 0 \\ 0 & -2 & 4 \\ 2 & 1 & 5 \\ 1 & 0 & 3 \end{bmatrix}.$$

If $A\mathbf{x}$ is defined, what is the dimension of the vector $\mathbf{x}$ and what is the dimension of $A\mathbf{x}$?

c. A vector whose entries are all zero is denoted by $\mathbf{0}$. If A is a matrix, what is the product $A\mathbf{0}$?

d. Suppose that $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the identity matrix and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Find the product $I\mathbf{x}$ and explain why I is called the identity matrix.

e. Suppose we write the matrix A in terms of its columns as

$$A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \end{bmatrix}.$$

If the vector $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$, what is the product $A\mathbf{e}_1$?

f. Suppose that

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}.$$

Is there a vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$?