Homework Two

Linear Algebra

College of the Atlantic

Due Friday, April 12, 2024

There are two parts to this assignment.

Part 1: WeBWorK. Do Homework 02 which you will find the WeBWorK page here: https://webwork-hosting.runestone.academy/webwork2/coa-feldman-es2048im-spring2024 I recommend doing the WeBWorK part of the homework first. This will enable you to benefit from WeBWorK's instant feedback before you do part two.

Part 2: Non WeBWorK Problems. Here are some instructions for how to submit this part of the assignment.

- Do the problems by hand using pencil (or pen) and paper. There is no need to type this assignment.
- If you like working on a tablet, go for it.
- Make a pdf scan of your work using genius scan or some similar scanning app. Please make the homework into a single pdf, not multiple pdfs.
- Submit the assignment on google classroom. Please don't email it to me. (Between my two classes I will be receiving around 45 assignments a week. Keeping track of them all in email is challenging.)

Homework Expectations and Guidelines

These problems were written by Matt Boelkins (Grand Valley State University), for use with David Austin's textbook Understanding Linear Algebra.

1. A given linear system has been represented by an augmented matrix and the process of Gaussian elimination has begun. The current state of the process is shown in the augmented matrix below.

   Execute the next ***three*** row operations of Gaussian elimination. Clearly state the three row operations you used, employing notation like we've been using in class, and show the sequence of matrices that results. In particular, your work should show at least two additional matrices beyond the one stated here.

$$\left[\begin{array}{rrr|r} 1 & -4 & -5 & 2 \\ 0 & 1 & 2 & -3 \\ 0 & 3 & 8 & 0 \end{array}\right]$$

2. Consider the linear system

$$\begin{array}{rcrcrcr} 2x_1 & - & 2x_2 & & & = & 10 \\ -3x_1 & & & - & 3x_3 & = & -9 \\ x_1 & + & 2x_2 & + & 3x_3 & = & -1 \end{array}$$

   Write the augmented matrix that corresponds to the given system. Then,

   (a) work by hand to find the reduced row echelon form of the augmented matrix,

   (b) identify which variables are basic and which are free, and

   (c) describe the solution space of the system, using a parametric description if there are infinitely many solutions.

3. Give an example of an augmented matrix in reduced row echelon form that describes a linear system having the stated properties. For each example you generate, (i) circle the pivot positions in the augmented matrix, and (ii) write one short sentence that explains why your example meets the stated criteria. If it is not possible to find such an example, explain why not.

   (a) a linear system having 3 equations and 5 unknowns and having no solutions.

   (b) a linear system having 3 equations and 5 unknowns and having infinitely many solutions.

   (c) a linear system having 3 equations and 5 unknowns and having exactly one solution.

   (d) a linear system having 4 equations and 2 unknowns and having no solutions.

   (e) a linear system having 4 equations and 2 unknowns and having infinitely many solutions.

   (f) a linear system having 4 equations and 2 unknowns and having exactly one solution.