Linearity

Linear Algebra

College of the Atlantic

1. Convince yourself that $R : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is linear, where R reflects vectors about the x axis.

2. Show that the derivative operator $\frac{d}{dx}$ is linear.

3. Let's now consider $\mathbb{P}_3$, the set of all cubic polynomials. These are functions that have the following form.

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ . \tag{1}$$

(using a two-observer spacetime diagram). Specifying a particular such function requires specifying the values of four numbers (a_0, a_1, a_2, a_3), which we can write as vector $\vec{y}$:

$$\vec{y} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \tag{2}$$

We'll now consider the derivative operator restricted to cubic polynomials: $\frac{d}{dx} : \mathbb{P}_3 \mapsto \mathbb{P}_3$. Write $\frac{d}{dx}$ as a matrix transformation.