Homework Assignment One

Complex Networks

College of the Atlantic

Fall 2008

Due Friday September 26, 2008.

Note: If you consult any books or web pages as you do these problems, that's fine, but please cite your sources.

1. This problem is designed to be straightforward practice calculating some key descriptive graph properties. For the graph in Fig. 1, determine the following:
   (a) The adjacency matrix.
   (b) Write the links down in list form.
   (c) The degree k_i of each node.
   (d) The average degree $\langle k \rangle$ for the entire graph.
   (e) The degree distribution $P(k)$.
   (f) The diameter of the graph.
   (g) The cluster coefficient C_i for each node.
   (h) The average cluster coefficient C.
   (i) The path length d_{ij} between nodes:
      i. 1 and 3
      ii. 2 and 8
      iii. 2 and 4
      iv. 6 and 1

   Use the second definition for the cluster coefficient, $C_i^{(2)}$ from the class notes. In your responses, be sure to state clearly the definitions of all quantities you're calculating.

2. Consider a regular graph in which the nodes are arranged linearly and every node is connected to its nearest and next-nearest neighbors. Such a graph is illustrated in Fig. 2. Assume that the graph has 100 total nodes. Also, assume that the graph wraps around so that it looks like a big circle. I.e., node 100 is connected to node 1.
   (a) Calculate the average cluster coefficient.

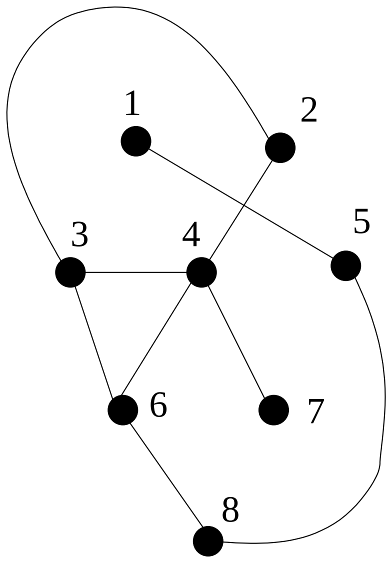

Figure 1: The network for problem 1.

(b) Calculate the diameter.
(c) Calculate the mean path length ℓ.
(d) Calculate the average degree $\langle k \rangle$.

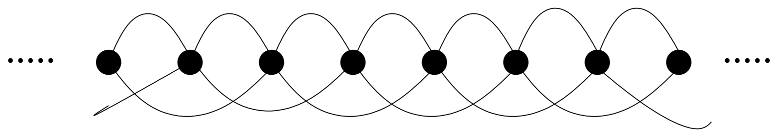

Figure 2: The network for problem 2.

3. Now consider again a graph of the form shown in Fig. 2. This time we'll consider a more general case. Let there be N total nodes in the graph.

   (a) Calculate the average cluster coefficient.
   (b) Calculate the diameter.
   (c) Calculate the average degree $\langle k \rangle$.
   (d) Calculate the mean path length ℓ.

(e) Is this a small-world graph? Why or why not?

4. Consider a very simple graph: eleven nodes connected together in a ring. Every node is connected only to its two nearest neighbors.

   (a) Calculate the mean distance ℓ between nodes.
   (b) Now, imagine that one additional link is added to the network at random. I.e., a node is chosen at random, and then a different node is chosen at random, and a link is drawn between those two nodes. What is the expected value for ℓ now?

5. **Optional.** (Recommended if you want to gain insight into small-world phenomena, and/or want a moderately challenging math problem.) Repeat the above problem, but for a the general case—i.e. for a ring of N nodes. You may assume that N is odd if you want.

6. The number of roadkill found along 100 mile stretches of interstate highway is distributed according to a Poisson distribution with a mean of 2.5.

   (a) What is the probability that there are 2 roadkill on a 100 mile segment of highway?
   (b) What is the probability that there are 3 roadkill on a 100 mile segment of highway?
   (c) What is the probability that there are 10 roadkill on a 100 mile segment of highway?
   (d) What is the probability that there are 4, 5, or 6 roadkill on a 100 mile segment of highway?

7. Consider an Erdős-Rényi model with $N = 100$ and $p = 0.05$.

   (a) What is the expected average degree?
   (b) What is the probability that a node has degree 10?

8. **Optional.** Do this if you like probability and/or want to mess around with limits. In class I asserted that the binomial distribution is well approximated by the Poisson distribution:

$$\binom{n}{k} p^k (1-p)^{n-k} \approx \frac{z^k e^{-z}}{k!} \,, \tag{1}$$

   where $z \equiv p(n-1)$. Show that Eq. (1) is true in limit of large n and fixed k. This is a standard but difficult calculation. It's not specific to random graphs but is a general result from probability theory. To show this you'll probably need to consult other references. If you do so, be sure to cite your sources and explain your method thoroughly.