Homework Assignment Two

Complex Networks

College of the Atlantic

Fall 2008

Due Friday 10 October, 2008.

Note: If you consult any books or web pages as you do these problems, that's fine, but please cite your sources.

1. In class, when solving the Barabasi-Albert preferential attachment model, I claimed that the function
$$k(t) = m\left(\frac{t}{t_i}\right)^{\frac{1}{2}} \tag{1}$$
was a solution to the differential equation:
$$\frac{dk}{dt} = \frac{k}{2t} \,. \tag{2}$$
Verify that this is the case by plugging Eq. (1) into Eq. (2).

2. Find a peer-reviewed or scholarly paper which claims that some variable or phenomena is distributed according to some distribution. Choose something interesting and fun. Write a paragraph or two which should include the full citation of the reference and address the following questions:
   (a) What evidence is given, if any, for the quantity being distributed as the authors claim?
   (b) How do the authors estiamte the parameters of the distribution? I.e., if the distribution is Poisson, how do they figure out λ?
   (c) Why, according to the authors, is this quantity interesting or important? Is there any significance to the particular distribution? What is their goal in modeling the phenomena via probability?

   Be prepared to briefly present and discuss your distribution *in class on Friday October 10.*

3. Consider the general power law distribution for a continuous variable x:

$$p(x) = kx^{-\alpha} \, , \tag{3}$$

where $x \geq 1$ and $\alpha \geq 0$.

   (a) Determine k.
   (b) Determine $\bar{x}$, the mean value of x.
   (c) Determine σ^2, the variance of x. Recall that $\sigma^2 = \bar{x^2} - \bar{x}^2$.
   (d) For which values of α do the above quantities exist?

4. Barrat and Weight [1] derived the following approximate expression for the cluster coefficient $C(\beta)$ of the Watts-Strogatz small world model:

$$C(\beta) \, = \, C(0)(1-\beta)^3 \, , \tag{4}$$

where β is the rewiring probability and $C(0)$ is the cluster coefficient when $\beta = 0$. Let's test this formula via simulation. Use the applet available at `http://ccl.northwestern.edu/netlogo/models/run.cgi?SmallWorlds.839.533`.

   (a) First, use the applet to calculate the cluster coefficient when the rewiring probability is zero.
   (b) Now, choose a non-zero β, have the program rewrire and record the cluster coefficient. Repeat this experiment a number of times (perhaps $n = 20$ or so), and calculate the average and standard deviation for $C(\beta)$. Are your results in agreement with Barrat and Weight's theory?
   (c) Repeat the experiment with the same β and the same number of trials n but with a larger number of nodes N in the network. What happens to the standard deviation in $C(\beta)$?

References

[1] A. Barrat and M. Weigt. On the properties of small-world network models. *The European Physical Journal B - Condensed Matter*, 13(3):547–560, January 2000.