Homework 6

Due Friday 20 February, 2004

Consider the following three-species coupled Lotka-Volterra system:

$$\frac{dx}{dt} = ax - bxy \ , \tag{1}$$

$$\frac{dy}{dt} = -cy + dxy - eyz \ , \tag{2}$$

$$\frac{dz}{dt} = -fz + gyz \ . \tag{3}$$

Here x, y, and z are populations, and a, b, c, d, e, f, and g are constants.

1. For each of the following sets of parameters, have Maple do the following:

   - Plot $x(t)$ vs. t
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   - Plot $x(t)$ vs. t
   - Plot the solution curve in the phase space $x(t)$, $y(t)$, $z(t)$. Note that the phase space is three dimensional!
   - Discuss in one or two sentences what's happening. How do you interpret the graphs? I'm not looking for any deep biology, just a statement like: x goes extinct while y and z cycle (or whatever).

   I'll send you a Maple worksheet that will help you get started.

   (a) $a = 1.0$, $b = 1.0$, $c = 1.0$, $d = 1.0$, $e = 1.0$, $f = 1.0$, $g = 1.0$

   (b) $a = 1.0$, $b = 1.0$, $c = 1.0$, $d = 1.0$, $e = 1.0$, $f = 1.0$, $g = 0.88$

   (c) $a = 0.5$, $b = 1.0$, $c = 1.0$, $d = 1.0$, $e = 1.0$, $f = 1.0$, $g = 1.0$

   (d) $a = 1.0$, $b = 1.0$, $c = 1.0$, $d = 1.0$, $e = 1.0$, $f = 1.0$, $g = 1.1$

2. Choose a few other parameter settings and see what happens.

3. Now modify the system as follows:

$$\frac{dx}{dt} = x - xy + Cx^2 - Azx^2 \ , \tag{4}$$

$$\frac{dy}{dt} = -y + xy \ , \tag{5}$$

$$\frac{dz}{dt} = -Bz + Azx^2 \ . \tag{6}$$

(a) Interpret the terms in the equation. (You may have to be somewhat creative.) Which species are predator, and which prey?

(b) Examine the behavior of this system for the following parameter values. For some values you may need to go out to very long times in order to see the final behavior.

i. $A = 1.0$, $B = 1.0$, $C = 1.0$

ii. $A = 2.9851$, $B = 3.0$, $C = 2.0$

iii. $A = 2.5$, $B = 3.0$, $C = 2.0$

4. Section 3.1, problem 5

5. Section 3.1, problem 6

6. Section 3.1, problem 8

7. Section 3.1, problem 9