

Differential Equations

Homework Three

Due 9 May 2008

1. Chapter 5, problem 6. (*Optional*: Come up with a situation from economics or business to which this model might apply.)
2. Chapter 5, problem 7. (*Optional*: Come up with a situation from economics or business to which this model might apply.)
3. Chapter 5, problem 9.
4. In this problem we will extend the Lotka-Volterra predator-prey model. Start with the basic model, Eq. (4) on page 123 of our textbook.
 - (a) Modify the prey growth term so that the growth is logistic and not exponential.
 - (b) Modify the predator growth term so that there is a threshold effect such that beyond a certain density of prey there is no further benefit to the predators. I.e., the predators have a finite appetite. There are a number of ways to take this into account in your model.
 - (c) Briefly explain the meaning or interpretation of all constants in your new model.
 - (d) Explain why your modified equation captures the threshold effect described above. A plot may be more useful than words.
 - (e) Check that your model makes sense by examining its limiting behavior. What happens if there are initially no predators? Or if there are no prey?
 - (f) Use Maple to analyze this model. Are there cycles? How many stable equilibria are there? Are there any bifurcations? To answer these questions, you'll probably want to produce both of species vs. time curves and phase space plots.