Differential Equations

Homework Five

Due 6 June 2008

This problem set is not yet complete. I'll be adding a few more problems Tuesday afternoon.

1. Consider an object of mass $m = 1$ attached to a spring with spring constant $k = 3$. Assume that the mass is acted on by a friction, or damping, force that is proportional to the object's velocity. I.e.,
$$F_d = -cv \,, \tag{1}$$
where c is a constant. For this problem, let $c = 2$. The resulting differential equation is
$$m\frac{d^2x}{dt^2} = -kx - c\frac{dx}{dt} \,. \tag{2}$$
   (a) Convert the above second-order equation into a coupled, linear, first-order system.
   (b) Solve the system of equations, using the initial values $x(0) = 4$, $x'(0) = 0$.
   (c) Sketch $x(t)$ vs. t and $v(t)$ vs. t. Do your answers make sense physically?
   (d) Sketch the $x(t), v(t)$ on the phase plane.

2. Here is another commonly used method to solve second-order equations with constant coefficients. Just guess a solution of the form $x(t) = e^{\gamma t}$. Plug this guess into the differential equation, and solve for γ. If guessing seems wishy-washy to you, you can call this an *ansatz*, which means "guess" or "approach" in German. (This is actually a standard mathematical term.) Anyway, try the ansatz $x(t) = e^{\gamma t}$ to solve Eq. (2).

3. *Optional, but strongly recommended for those interested in physics and engineering. Some algebra, but not that difficult.* Consider again the second-order equation for a damped oscillator, Eq. (2).
   (a) Re-write the equation in terms of ω and ξ, where
$$\omega = \sqrt{\frac{k}{m}} \tag{3}$$
   and
$$\xi = \frac{c}{2\sqrt{km}} \,, \tag{4}$$
   (b) Convert to a coupled, first-order equation and find the eigenvalues for the matrix.
   (c) You should notice a bifurcation in the behavior of the solutions. Characterize the two types of solutions that are possible. Feel free to consult a reference on damped harmonic oscillators; wikipedia has a good explanation.

4. Chapter 9, problem 1. Please be sure to give units for all your answers.

5. Chapter 9, problem 2.
6. Chapter 10, problem 2. *Optional. Recommended for beverage fans.*
7. Chapter 11, problem 2.
8. Chapter 11, problem 11. *Optional. Recommended for Steve Ressel fans.*
9. Chapter 11, problem 13. *Optional. Ever wonder why you would ever need hyperbolic trigonometric functions? This problem will sate your curiosity.*