Differential Equations
Homework One
Due Friday, September 26, 2014

In this assignment you will carry out an analysis of the logistic differential equation:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) . \tag{1}$$

This equation describes how a population $P(t)$ varies over time. In the equation k and N are parameters. You will analyze this in several different but complementary ways, much we have done with Newton's law of cooling in class. We will restrict our analysis to non-negative values of P.

1. Sketch the right-hand side of Eq. (1). Use $k = 2$ and $N = 500$.
2. Make qualitatively accurate sketches of $P(t)$ for initial populations of 100, 400, and 700.
3. What is the biological significance of the values of k and N in the equation? (I'm just looking for one or two sentences, not a lengthy exegesis.)
4. Verify that the following is a solution to Eq. (1):

   $$P(t) = \frac{NP_0}{P_0 + (N - P_0)e^{-kt}} , \tag{2}$$

   where P_0 is the initial population. To do so, you'll need to plug the above expression into Eq. (1) and show that the equation is true. It'll involve a bit of differentiation and algebra.
5. Use python to plot Eq. (2) for the three values of P_0 that you used in your sketches for Question 2.
6. Write a program that implements Euler's method for solving ordinary differential equations. Your program should be clearly written and be well commented.
7. Use your Euler program to make plots of your Euler solution for $\Delta t = 2$ and $\Delta t = 1$ from $t = 0$ to 10. Make plots of these two solutions together with the exact solution, Eq. (2). Use $P_0 = 100$.
8. Use your Euler program to experiment with different values of Δt. How small a Δt is small enough? Briefly explain.
9. Use your Euler program to plot solutions to Eq. (1) for the three P_0 values you used in Question 2. Plot all three solutions on the same axes. (Note that you've now made this same plot three different ways: a qualitative sketch by hand, an exact plot using python and a formula, and an essentially exact plot using python and Euler's method.)