Differential Equations
Homework Two
Due Friday, October 10, 2014

1. Consider the initial-value problem

$$\frac{dy}{dt} = 3y^{\frac{2}{3}}\ , y(0) = 0\ . \tag{1}$$

   (a) Show that $y_1(t) = 0$ is a solution to Eq. (1).
   (b) Show that $y_2(t) = t^3$ is a solution to Eq. (1).
   (c) On the same axes, make a rough sketch of $y_1(t)$ and $y_2(t)$.

2. The Lotka–Volterra equations are:

$$\frac{dR}{dt} = AR - BRF\ , \tag{2}$$

$$\frac{dF}{dt} = CRF - DF\ . \tag{3}$$

We will use the parameter values $A = 2$, $B = 0.5$, $C = 0.2$, and $D = 1$. Using my SIR model code (or starting from scratch), write a program that will solve the LV model (Eqs. (2) and (3)) and produce plots of R vs. t, F vs. t and the trajectory in phase space: a plot of F vs. R. There is no need to hand in this code.

   (a) Suppose we change Eq. (2) to:

$$\frac{dR}{dt} = AR\left(1 - \frac{R}{N}\right) - BRF\ . \tag{4}$$

   Biologically, what does this mean? What is the meaning of N?
   (b) Modify your code so that it solves the modified system (i.e, Eqs. (4) and (3). Use $N = 20$. What behavior do you observe? Try several different initial conditions? Include printouts of a few plots.
   (c) Double the value of C. How does this change the long-run behavior of the rabbit and fox populations? Briefly explain why your results make sense.
   (d) Return C to its original value but now make $N = 2000$. What long-term behavior do you observe? Explain.
   (e) Let's modify the equations further to allow for some nonlinearity in the rabbit-fox interaction:

$$\frac{dR}{dt} = AR\left(1 - \frac{R}{N}\right) - BRF^{\alpha}\ , \tag{5}$$

$$\frac{dF}{dt} = CRF^{\alpha} - DF\ , \tag{6}$$

   where α is a parameter that controls the nonlinearity.

   i. Set $\alpha = 1.1$. What is the long-term behavior of the populations?
   ii. Set $\alpha = 1.5$. What is the long-term behavior of the populations?

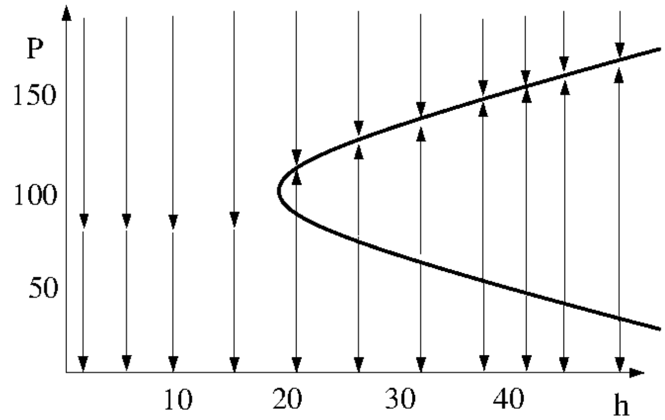

Figure 1: A bifurcation diagram.

3. Figure 1 shows the bifurcation diagram for a differential equation (not the logistic equation with harvest).
   (a) Sketch the phase line for the system for $h = 10$.
   (b) Sketch the phase line for the system for $h = 30$.
   (c) Sketch the phase line for the system for $h = 40$.
4. Consider the differential equation:

$$\frac{dy}{dt} = ry - y^3 \ . \tag{7}$$

   For each of the following r values, sketch the right-hand side of Eq. (7) (using WolframAlpha if you wish) and draw the phase line for y.
   (a) $r = -1$
   (b) $r = 0$
   (c) $r = 1$
5. Sketch a bifurcation diagram for Eq. (7).