Chapter C13: Angular Momentum

C13.3: Cross Product

The cross product is, like the dot product, a way to "multiply" two vectors together. Note that "·" and "×" do not mean the same thing!

C13.4 The Angular Momentum of a Particle

- An object's angular momentum relative to the point O is defined as:

$$\vec{L} \equiv m(\vec{r} \times \vec{v}) \,, \tag{1}$$

  where $\vec{v}$ is the object's velocity and $\vec{r}$ is a vector from point O to the object.

- Note that the particle need not be moving in a circle for it to have angular momentum.

- An easier way to think of the above formula: $L = mrv_\perp$, where $v_\perp$ is the component of $\vec{v}$ perpendicular to $\vec{r}$.

- However, if the particle is moving in a circle, then

$$\vec{L} = mr^2\vec{\omega} \tag{2}$$

C13.5 The Angular Momentum of a Rigid Object

For a solid object of moment of inertia I rotating with an angular velocity of $\vec{\omega}$,

$$\vec{L} = I\vec{\omega} \,. \tag{3}$$

If we have an object rotating about its axis and the object is also moving as a whole around a point O, then

$$\vec{L} = \vec{L}^{\mathrm{cm}} + \vec{L}^{\mathrm{rot}} \tag{4}$$

Figure 1:

Figure 2:

C13.6 Conservation of Angular Momentum

If a system is isolated, its angular momentum is conserved.

Examples:

1. You are standing at point X in Fig. 1. A 2 kg bird flies by you. What is its angular momentum with respect to point X when the bird is at point A? What is its angular momentum at point B?

2. A 20 kg child runs at 8 m/s and jumps on a 200 kg merry-go-round as shown in Fig. 2. What is the angular speed of the merry-go-round after the collision? The radius of the merry-go-round is 1.5 meters.

Practice:

1. A 5 kg serving tray with a radius of 20 cm spins at 5 revolutions per minute. A 1 kg coffee cup is placed on the tray 10 cm from the axis of rotation. What is its angular velocity now?

2. A spherical planet is spinning in outer space. The planet suddenly collapses so that its radius is half of what it was before. What is its new angular velocity compared to its old angular velocity?